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A Fuzzy World

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1.1 Introduction

> Two roads diverged in a wood
> And I took the one less traveled by
> And that has made all the difference.
> —Robert Frost, *The Road Less Traveled*

According to some, the only thing quantum theory has going for it is that it is unquestionably correct, as Michio Kaku remarked. Correct in the sense that if you give me the actual outcome of one measurement, it allows me to give you the correct probabilities of the possible outcomes of another measurement. The testable part of the theory concerns statistical correlations between measurement outcomes. This is the gist of what Redhead (1987) has called the "minimal instrumentalist interpretation." It is an interpretation in that it sets up a correspondence between the mathematical formalism and physical quantities. It is minimal in that it makes minimal reference to the physical world. And it is instrumentalist in that it identifies measurements as the interface between the mathematics and the physical world.

There is a second sense of "interpretation," different from that employed so far. This is "some account of the nature of the external world and/or our epistemological relation to it that serves to *explain* how it is that the statistical regularities predicted by the formalism with the minimal instrumentalist interpretation [first sense] come out the way they do." (Redhead, 1987) The expression "minimal instrumentalist interpretation" can also be used with this sense in mind. It then implies the futility of the search for such an account.

The quest for an interpretation in the second sense may be pursued along either of two lines. The first aims to interpret (some of) the theory's mathematical symbols as more or less directly representing physical entities or states. The second aims to uncover the ontological or epistemological implications of the theory's peculiar probability assignments. The first aims to divest measurements of their special status. The second aims to understand the reason or reasons for their special status. In what follows we shall pursue the second line of inquiry—the "road less traveled."

1.2 Core Rules

Here is one possible formulation of the core rules of the quantum-mechanical probability calculus. Suppose that we want to calculate the probability of a particular outcome of a measurement M_2 given the outcome of an earlier measurement M_1. To do this, we choose a sequence of measurements that may be made in the meantime, assign to each possible sequence of intermediate outcomes ("alternative") a complex number ("amplitude"), and apply the appropriate rule:

(A) If the intermediate measurements are made (or if it is possible to find out what their outcomes would have been if they had been made), first square the absolute values of the amplitudes of the alternatives and then add the results.

(B) If the intermediate measurements are not made (and if it is impossible to find out what their outcomes would have been if they had been made), first add the amplitudes of the alternatives and then square the absolute value of the result.

1.3 Quantum-Mechanical Probabilities Are Conditional

What is immediately obvious from the foregoing is that quantum-mechanical probabilities are *conditional probabilities* (Primas, 2003). The propagator $\langle x_f, t_f | x_i, t_i \rangle$, in particular, is the amplitude associated with the *conditional* probability of finding a given system at the point[1] x_f of its con-

[1] What is meant is of course the probability of detecting the system under consideration in a finite region R of its configuration space divided by the "volume" $V(R)$ in the limit

figuration space, *if* the appropriate measurement is made at the time t_f, *if* the previous measurement was made at t_i, and *if* this had the outcome x_i.

1.4 The Principle of Evolution

It is customary not only to introduce a "wave function" $\psi(x,t)$ such that

$$\psi(x_f, t_f) = \int dx_i \, \langle x_f, t_f | x_i, t_i \rangle \, \psi(x_i, t_i), \tag{1.1}$$

but also to regard the wave function as in some sense more fundamental than the propagator.[2] Even in the Bayesian approach to quantum mechanics, in which quantum states represent "degrees of belief" (Caves *et al.*, 2002, 2007), the evolution of quantum states take center stage. Why?

One reason no doubt is the historical precedence of Schrödinger's "wave mechanics" over Feynman's propagator-based formulation of the theory. A less obvious but perhaps more insidious reason may be the primacy of absolute over conditional probabilities in Kolmogorov's (1950) mathematical foundation of probability theory.[3] But the root cause of that notion appears to lie in what we may call the "principle of evolution," according to which physics can be neatly divided into kinematics and dynamics, the former concerned with the description of a physical system at any one time (its state), the latter concerned with the time evolution of the state. This principle is a hangover from classical times, when it seemed possible to transmogrify a time dependent algorithm or mathematical symbol (e.g., the Faraday tensor) into a physical entity that persists in time and changes with time (e.g., the electromagnetic field).

This sleight of hand no longer works, for a variety of reasons, not least among them the pseudo-problems created by previous attempts to keep it working (Mohrhoff, 2006a). Here I wish to emphasize two other reasons.

$V(R) \to 0$.

[2] In the case of a nonrelativistic system comprising n particles, x stands for a set of $3n$ coordinates, and $\psi(x,t)$ encapsulates the probabilities of the possible outcomes of any measurement to which the system may be subjected at the time t. Relativistically, particle number is a system variable, and $\psi(x,t)$—now technically an operator—defines a joint probability distribution over (i) all sets of particles that can be detected at the time t, and (ii) all outcomes that can be obtained by measurements performed on the detected set of particles.

[3] This too may be a fortuitous case of historical precedence. An axiomatic alternative to Kolmogorov's formulation of probability theory was developed by Rényi (1950, 1970). Every result of Kolmogorov's theory can be translated into Rényi's, which is based entirely on conditional probabilities (Primas, 2003).

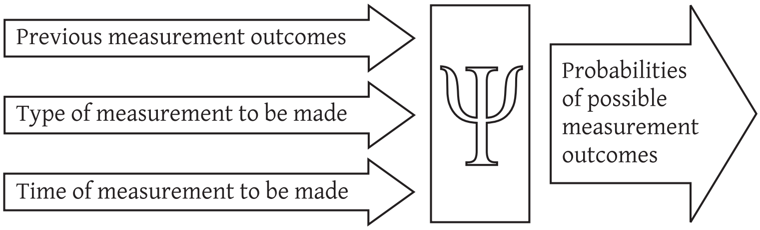

Fig. 1.1 Inputs and outputs of a quantum "state."

The first is that the time dependence of a quantum "state" ψ is a dependence on the time of the measurement to the possible outcomes of which ψ serves to assign probabilities (Fig. 1.1). It is *not* the time dependence of an evolving state of affairs. The second is an implication, discussed in Sec. 1.10, of the peculiar manner in which quantum mechanics assigns probabilities: neither the spatial nor the temporal differentiation of the physical world goes "all the way down." Because the physical world is completely differentiated neither spacewise nor timewise, there is no instant at which a physical state could obtain. There is no *evolving* physical state because this would exist at *each* instant of time.

1.5 Interpretational Strategy

To go beyond the minimal instrumentalist interpretation (first sense), we need to decide on an interpretational strategy. Do we want to keep determinism? Do we want the world to remain "contained" in an intrinsically and completely differentiated space, time, or spacetime? Do we want sharp particle trajectories? No matter which strategy we adopt, we must bear in mind that no interpretation (second sense) can be true (or false) the way the mathematical formalism together with the minimal instrumentalist interpretation (first sense) is true (or false).[4] Any account of what happens or obtains *between* measurements (in both the spatial and the temporal sense) or *over and above* measurements, is untestable by definition. The only truth criterion for such an account is whether we like it or not. It's as simple as that.[5] One might counter that the truth criterion is rather parsi-

[4]So far our only reason to doubt that the latter is true is our failure to make sense of it, but this appears to bear more on us than on the theory (Mohrhoff, 2007).

[5]After the motto "offense is the best defense," I submit the following. Researchers at Johns Hopkins, M.I.T., and other well-regarded universities inside and outside the U.S.A. have documented that "students who receive honor grades in college-level physics courses

mony, credibility, sensibleness or, at the very least, sanity, but let's not fool ourselves. What is parsimony to me may be extravagance to you, what I find credible you may find incredible, and what I regard as the epitome of sanity you may look on as the height of insanity.

Here, then, is the assumption that defines my interpretational strategy:

> Whenever quantum mechanics instructs us to add amplitudes rather than probabilities (i.e., to apply Rule B rather than Rule A), the distinctions we make between the alternatives correspond to nothing; they don't exist in the actual world; they exist solely in our heads.

1.6 A Scattering Experiment

As a first application, consider the following setup. Initially there are two incoming particles, one (N) heading northward and one (S) heading southward. We want to calculate the probability $p_\perp$ with which the particles scatter at right angles, so that in the end there is one particle (E) heading in a direction we shall call "eastward" and one particle (W) heading "westward." (The tacit assumption here is that the total momentum is zero, and that the two particles scatter elastically.) If it is possible to infer from any actual event or state of affairs whether S is the same particle as W (and thus N the same as E) or S is the same particle as E (and thus N the same

are frequently unable to solve basic problems and questions encountered in a form slightly different from that on which they have been formally instructed and tested.... Indeed, in dozens of studies of this sort, young adults trained in science continue to exhibit the very same misconceptions and misunderstandings that one encounters in primary school children." (Gardner, 1991) A regularly updated bibliography (Duit, 2007), which presently has about 7,700 entries documenting change in students' conceptions, demonstrates that little or no change happens when students experience even the best of standard science instruction. Why is this the case? Part of the answer is the orthodox view that once the teacher has presented the established canon by approved methods, she has done her job. Whether or not a student "gets" it is out of her hands. As a result, what most science students learn is that they are on the lower rung of a system in which they are dependent on a higher rung for declarations of the truth. Science instruction thus fails society by promoting elitism and rendering most of its members intellectually stunted. Why then are alternative approaches to teaching science, which have been demonstrated to yield *significant progress* in understanding (Dykstra, 2005), resisted or ignored? Because in substituting a *constructivist* epistemology (Von Glasersfeld, 1991) for the *realist* conception of the knowledge constituting the canon, these alternative approaches deflate the status of the institution of science by controverting if not an explicit claim of access to ontological truth then certainly the carefully cultivated perception of such access.

as W), then Rule A applies,

$$p_\perp = |A_1|^2 + |A_2|^2, \tag{1.2}$$

where A_1 and A_2 are the amplitudes associated with the alternatives. This is just the classical rule according to which an event that can come to pass in either of two ways, with respective probabilities p_1 and p_2, comes to pass with probability

$$p = p_1 + p_2. \tag{1.3}$$

If there isn't any actual event or state of affairs from which one could infer whether S is the same as W or the same as E, then Rule B applies:[6]

$$p_\perp = |A_1 + A_2|^2. \tag{1.4}$$

If there are no preferred directions, then for bosons we have $A_2 = A_1$ whereas for fermions we have $A_2 = -A_1$. Hence for bosons of the same type, $p_\perp$ is twice as much as it is for distinguishable particles, whereas for fermions of the same type it is zero.

1.7 There Only Is Room For One

Whenever either of the alternatives takes place, the classical rule (1.3) holds. Hence if it does not hold, *neither* alternative takes place (to the exclusion of the other), and so neither of the outgoing particles is identical with either of the incoming particles (to the exclusion of the other). The question "Which incoming particle is identical with which outgoing particle?" is therefore meaningless, not intrinsically, but because it arises from a false assumption—the assumption that either of the alternatives takes place.[7]

Here as elsewhere, the challenge is to learn to think in ways that do not lead to meaningless questions. The question "Which is which?" arises because we assume that initially there are two things, one moving northward and one moving southward, that in the end there are two things, one moving eastward and one moving westward, and that each of these things remains identical with itself. To obviate the question, we must assume that initially there is *one* thing moving *both* northward and southward, and that in the end there is *one* thing moving *both* eastward and westward.

[6] The antecedent rules out, *inter alia*, that the incoming particles are of different types.

[7] Since what is essential to the present argument is the statistics of indistinguishable quantum systems, any pair of such systems and any two sets of initial and final measurement outcomes would have led to the same conclusion.

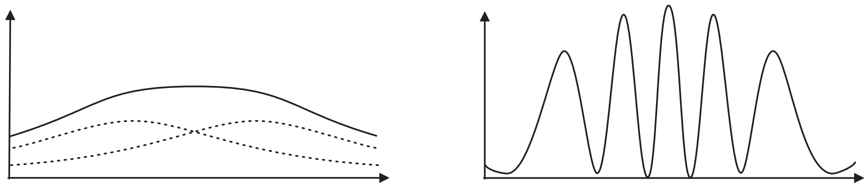

Fig. 1.2 Probability of detection across the backdrop, calculated according to Rule A (left) and according to Rule B (right).

Starting directly from our chosen interpretational strategy, we reach the same conclusion. We are to think in such a way that the two alternatives cease to be distinct. They appear to be distinct because we tend to conceive of them in terms of distinct, persistent entities. They cease to be distinct as soon as we cease to think in these terms. But this means that ultimately the physical world has room for one persistent entity only. The differences that exist are differences between properties, all of which belong to the same entity.

1.8 A Two-Slit Experiment

The two-slit experiment with electrons is too well know to require a description. The initial measurement indicates the position at which an electron was launched or, if this is known, the fact that an electron was launched. The intermediate measurement, which may or may not be made, indicates the slit that was taken by an electron. The final measurement, the probabilities of the possible outcomes of which we wish to calculate, indicates the position (along an axis across the backdrop) at which an electron was detected. The familiar probability distributions obtained under the conditions stipulated by the two core rules are plotted in Fig. 1.2.

The distribution calculated according to Rule A is the sum of two distributions, one ($p_1(x)$, the dotted curve with a peak in the left half of the diagram) for electrons that were found passing through the left slit (L) and one ($p_2(x)$, the dotted curve with a peak in the right half of the diagram) for electrons that were found passing through the right slit (R), in agreement with Eq. (1.3). As Bohm (1952) has shown, (1.3) may also hold under the conditions stipulated by Rule B, if the state of one slit (open or shut) is allowed to influence the electrons that pass through the other slit—i.e., if $p_1(x)$ and $p_2(x)$ are allowed to depend on the state of R and L, respectively.

Our own interpretational strategy requires us instead to think in such a way that the two alternatives cease to be distinct. But if it is not the case that each electron goes through a particular slit, then it must be the case that in some sense each electron goes through *both* slits. However, the sense of "the electron went through both slits" cannot be the conjunction

c = "The electron went through L" and "the electron went through R".

To establish the truth of **c**, we must find that the electron went through L *and* that it went through R. This never happens. The only possible meaning of "an electron went through both slits" is that it went through L&R—the cutouts in the slit plate considered *as a whole*—without going through a particular slit *and* without having parts that go through different slits.

1.9 Spatial Distinctions: Relative and Contingent

And so the reality of spatial distinctions is *relative*: the distinction we make between two regions or paths may be real for one object at one time and not real for a different object at the same time or for the same object at a different time.[8] Saying that a spatial distinction is "real for" a given object is another way of saying that this object went through, or followed, either alternative but not both.

If spatial distinctions are relative, then they are also *contingent*. Whether or not a given distinction is real for a given object (at a given time) depends on what is measured. Specifically, if the slit taken by a particular electron is indicated (by a measurement, needless to say), then the distinction we make between the slits is real for the electron, otherwise it is not.[9] If we think this through, we arrive at the conclusion that a position is possessed only if, only when, and only to the extent that its possession is indicated. The distinctions we make between the regions defined by a

[8]For example, if a dim source of light is placed behind the slit plate, an electron may or may not scatter a photon as it emerges from the slits. A scattered photon can provide information about where it was scattered, and hence about the slit taken by the electron that scattered it. If two electrons pass through the slits but only one of them scatters a photon, then one electron went through a particular slit while the other electron went through both slits in the sense just explicated. In this case the distinction between L and R is real for one electron but not for the other.

[9]Display of interference phenomena is sufficient but not necessary for a distinction between two alternatives to lack objective reality. As "which way" experiments illustrate (Scully *et al.*, 1991; Englert *et al.*, 1994; Mohrhoff, 1999), such a distinction can lack objective reality even in the absence of interference phenomena. This is why it is possible to "restore interference."

partition are real for a given object only if (and only when) this is subjected to a position measurement capable of distinguishing between these regions.

If the reality of spatial distinctions is relative and contingent, then physical space cannot be something that has parts. For consider a partition of space. If the regions defined by this partition were intrinsic to it, and thus by themselves distinct, the distinctions we make between them would be real *per se* and thus necessarily real for every object in space. It follows that a detector[10] is needed not only to indicate the presence of an object in its sensitive region V but also, and in the first place, to realize (make real) a region V of space—or the distinction between being inside V and being outside V—and thereby make the predicates "inside V" and "outside V" available for attribution.

This bears generalization. The measurement apparatus that is presupposed by every quantum-mechanical probability assignment is needed not only for the purpose of indicating the possession of a particular property or value but also, and in the first place, for the purpose of realizing a set of attributable properties or values. While this amply justifies Bohr's insistence that out of relation to experimental arrangements the properties of quantum systems are undefined (Petersen, 1968), our interpretational strategy takes us further, for we are in a position to see why this is so. Because physical space is intrinsically undifferentiated, it falls to its "content" to realize spatial distinctions and the properties that these serve to define.

But suppose that W is a region disjoint from V, and that the presence of an object O in V is indicated. Is not O's absence from W indicated as a consequence? Are we not entitled to infer from this that "O is in W" is false?

Because the reality of spatial distinctions is relative and contingent, the answer is negative. Regions of space do not exist "by themselves." The distinction we make between "inside W" and "outside W" has no reality *per se*. If W is not realized by being the sensitive region of a detector in the broadest sense of the word—anything capable of indicating the presence of something somewhere—then it does not exist, and if it does not exist, then the proposition "O is in W" cannot be in possession of a truth value. Neither the property of being inside W nor the property of being outside W is available for attribution to O. All we can infer from O's indicated presence in V is the truth of a counterfactual: if W *were* the sensitive region of a

[10]A perfect detector, to be precise; if a detector for particles of type X is less than 100% efficient, then the absence of a click does not warrant the absence of a particle of type X from its sensitive region.

detector D, then O *would* not be detected by D.

1.10 Spatial Distinctions: Not All the Way Down

Let $I\!R^3(\mathrm{O})$ be the set of exact positions relative to an object O. Since no material object ever has a sharp position,[11] we can conceive of a partition of $I\!R^3(\mathrm{O})$ into finite regions that are so small that none of them is the sensitive region of an actually existing detector. Hence we can conceive of a partition of $I\!R^3(\mathrm{O})$ into sufficiently small but finite regions V_k of which the following is true: there is no object Q and no region V_k such that the proposition "Q is inside V_k" has a truth value. In other words, there is no object Q and no region V_k such that V_k exists for Q. But a region of space that does not exist for any material object, does not exist at all. The regions V_k correspond to nothing in the physical world. They exist solely in our heads. Bottom line: the spatial differentiation of the physical world is incomplete—it doesn't go all the way down.

As the world's incomplete spatial differentiation follows from the nonexistence of exact positions, so its incomplete temporal differentiation follows from the nonexistence of exact times. (The times at which observables possess values, like the possessed values themselves, must be indicated in order to exist. Clocks are needed not only to indicate time but also, and in the first place, to make times available for attribution to indicated values. Since clocks indicate times by the positions of their hands,[12] and since sharp positions do not exist, neither do exact times.)

1.11 Fuzzy Observables

"Ordinary objects" are composed of a large but finite number of unextended objects (particles that do not "occupy" space), they "occupy" finite volumes of space, and they are stable: they neither explode nor collapse the moment they are created. Thanks to the quantum theory, we know that the

[11]In a nonrelativistic world, this is so because the exact localization of a particle implies an infinite momentum dispersion and thus an infinite mean energy. In a relativistic world, the attempt to produce a strictly localized particle results instead in the creation of particle-antiparticle pairs. For more on this subject see the next section.

[12]Digital clocks indicate times by transitions from one reading to another, without hands. The uncertainty principle for energy and time, however, implies that such a transition cannot occur at an exact time, except in the unphysical limit of infinite mean energy (Hilgevoord, 1998).

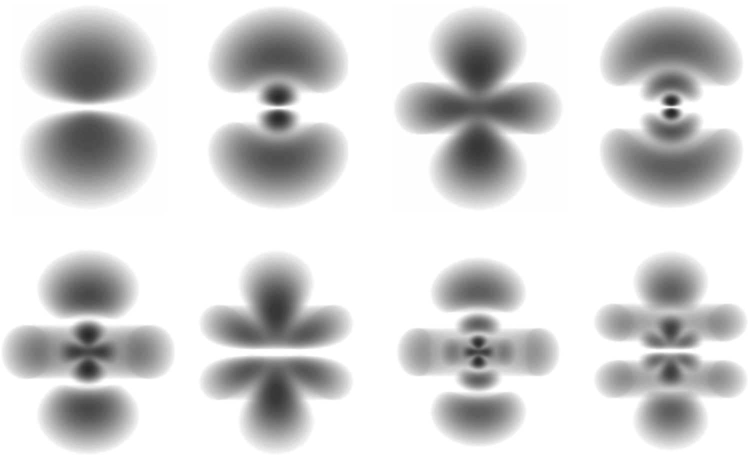

Fig. 1.3 Fuzzy electron positions relative to the nucleus in stationary states of atomic hydrogen.

existence of such objects hinges on the *fuzziness* of the relative positions and momenta of their constituents.[13] This, rather than our subjective "uncertainty" about the values of these observables, is what "fluffs out" matter. (Heisenberg did not speak of "uncertainty." The actual meaning of his term *Unschärfe* is "fuzziness.")

How does one define and quantify a fuzzy observable? By assigning probabilities to the possible outcomes of a measurement![14] To be precise, by making *counterfactual* probability assignments. To see why, imagine a small bounded region V inside one of the "position probability clouds" in Fig. 1.3. As long as the atom is in one of the corresponding stationary states,[15] the electron is neither inside V nor outside. (If it were inside, the probability of finding it there would be 1, and if it were outside, the probability of finding it there would be 0, neither of which is correct.) If on the other hand we

[13]The fuzziness of the relative momenta is counterbalanced by the electrostatic attraction that binds the components. The attraction alone would cause the "position probability clouds" in Fig. 1.3 to shrink; the fuzziness of the electron's momentum relative to the nucleus alone would cause them to expand.

[14]This goes some way towards explaining why the mathematical formalism of the theory is a probability calculus, and why measurements have the special status that they do.

[15]Quantum states are probability algorithms. If they are stationary, the probabilities they assign do not depend on the time of the measurement to the possible outcomes of which they are assigned, but all quantum states depend on actual measurement outcomes, in this case the outcomes of three measurements: the atom's energy, its total angular momentum, and one component of its angular momentum.

were to determine (by making the appropriate measurement) whether the electron is inside or outside V, we would find it either inside or outside V. Thus in order to quantitatively describe a fuzzy observable, we must assume that a measurement is made, and if we do not want our description to change the observable described, we must assume that no measurement is made. In other words, we must describe it by assigning probabilities to the possible outcomes of *unperformed* measurements.

1.12 The Shapes of Things

There is one notion that is decidedly at odds with the incomplete spatial differentiation of the physical world (Sec. 1.13). It is the notion that the "ultimate constituents" of matter are pointlike or (God help us) stringlike (Green *et al.*, 1988). This raises the question of what we actually mean by the *form* of a fundamental particle—a particle without internal structure.[16] A sensible definition of the form of a *composite* object would be: the relative positions and orientations between its components, together with the forms of the components. If the components are fundamental particles, the definition ceases to be recursive, and we are confronted with the above question.

What does the theory say on this issue? Nothing! Nothing in the formalism of quantum mechanics refers to the shape of an object lacking internal structure. And experiments? While they can provide evidence of internal structure, they cannot provide evidence of absence of internal structure. *A fortiori*, they cannot provide evidence of a pointlike form, inasmuch as this would be evidence of absence of internal structure. The notion that an object without internal structure has a pointlike form—or any other form for that matter—is therefore unwarranted on both theoretical and experimental grounds. In addition, it explains nothing. Specifically, it does not explain why a composite object—be it a nucleon, a molecule, or a galaxy—has the shape that it does, since all empirically accessible forms are fully accounted for by the relative positions and orientations of their material constituents. All that that notion does is encumber our efforts to make sense of the quantum world with a type of form whose existence is completely unverifiable, which is explanatorily completely useless, and which differs radically from all empirically accessible forms, inasmuch as these are sets of spatial relations (relative positions and orientations). We

[16]If we stick to the well-established standard model, this means quarks and leptons.

shall therefore commit it unceremoniously to the realm of the nonexistent.

Thus the form of a composite object ultimately consists of the relative positions between formless "constituent parts," and the "ultimate constituents of matter" are formless.

1.13 Space

Now we are ready to address the question: what is space? But let's put it less ambitiously: what would be an appropriate way of thinking about the spatial aspect of the physical world? We tend to think of space as a self-existent expanse, which "contains" matter. But as we concluded in Sec. 1.9, if space is such an expanse, then it cannot have parts.

Take another look at Fig. 1.3. What we see is neither the (formless) electron nor the nucleus. We see the electron's fuzzy position relative to the nucleus in various states of atomic hydrogen. And we see an expanse over which the electron's position is "smeared out." Does this expanse have parts? If it had, the electron's position itself would have parts. But this makes no sense; one can divide an object, and thereby create as many positions as there are parts (or create as many relative positions as there are pairs of parts), but one cannot divide a position. This expanse, therefore, lacks parts.

Does it contain matter? Certainly not in the everyday sense of containment, since an everyday container has a boundary, whereas this expanse has none. Does space—in case it's something else—contain matter? By the same token, not in the everyday sense. If space contains anything—in the proper, set-theoretic sense of containment—it is positions and orientations or, rather, *relative* positions and *relative* orientations. It contains spatial relations. Space may also be said to contain the forms of material objects, but this doesn't add anything new, since the forms of material objects *are* sets of spatial relations. Space contains the forms of all things that have forms, but it does not contain matter, inasmuch as it does not contain the "ultimate constituents of matter." It contains spatial relations, but it does not contain their relata. Instead, it exists "between" them; it is the "web" spun by their relations.[17]

If space is a set of spatial relations, then what about the expanse we see

[17]This view is close to relationism, the doctrine that space and time are a family of spatial and temporal relations holding among the material constituents of the universe. It has been claimed that relationism fails to accommodate inertial effects. See Dieks (2001a,b) for a refutation of this claim.

in Fig. 1.3? We may think of it as a self-existent (substantial) expanse, to which all spatial relations owe their spatial quality (without which distances would be just numbers)—provided that we also think of it as undifferentiated: it lacks parts. However, it would be more parsimonious not to introduce such an expanse *over and above* the quality of extension that all spatial relations share. There is no need for a substantial expanse to which they *owe* this quality. The intrinsically undifferentiated expanse we seen in Fig. 1.3 (and everywhere else) *is* this quality.

1.14 The Macroworld

Much effort has been devoted to explaining the emergence of classical behavior (Pearle, 1979; Joos and Zeh, 1985; Ghirardi *et al.*, 1986; Joos *et al.*, 2003; Zurek, 2003). Why is classical physics an acceptable approximate theory—to the extent it is? And first of all, if a macroscopic or classical domain emerges, from where does it emerge? Our interpretational strategy has led us to conclude that the physical world has room for a single persistent entity only (Sec. 1.7). The ultimate possessor of all existing physical properties, it is intrinsically without properties. And like the fundamental particles it constitutes, it is formless. This, rather than a probability algorithm or some mathematical symbol—transmogrified into a physical process or entity—is the matrix from which the familiar "classical world" emerges. Needless to say, there is no fully classical world. But there is an emergent macroworld, in which the property-indicating events that are correlated via the quantum laws take place.

To arrive at a consistent way of thinking about the macroworld, we first observe that in a world in which all relative positions are more or less fuzzy, some objects have the sharpest positions in existence. For these objects, the probability of obtaining evidence of departures from their classically predictable positions is necessarily very low. Hence *among* these objects there will be many of which the following is true: every one of their indicated positions is consistent with (i) every prediction that could in principle be made on the basis of their previously indicated positions and (ii) a classical law of motion. These are the objects that deserve the label "macroscopic".[18] To enable a macroscopic object to indicate an unpredictable value—to func-

[18]This definition does not require that the probability of finding a macroscopic object where classically it could not be, is strictly zero. What it requires is that there be no position-indicating event that is inconsistent with predictions that could in principle be made on the basis of a classical law of motion and earlier position-indicating events.

tion as the proverbial "pointer"—one exception has to be made: its position may change unpredictably if and when it serves to indicate such a value. Instead of being evidence of the fuzziness of the value-indicating position, such an unpredictable change is evidence of the fuzziness of the observable measured—the fuzziness that would have obtained if no measurement had been made (Sec. 1.11).

1.15 The Emergence of the Macroworld

We know now what emerges, and we know whence it emerges. So *how* does it emerge? Since all that space contains is (i) spatial relations between formless relata and (ii) forms consisting of spatial relations (Sec. 1.13), all that we need to account for is the coming into being (i) of spatial relations and (ii) of forms.

Here is how we get both matter and space out of a single persistent entity (Sec. 1.7): by entering into spatial relations with itself, this single persistent entity creates the multitude of spatial relations that constitute space, and it creates the apparent multitude of relata (a.k.a. fundamental particles) that constitute matter—apparent because the relations are *self*-relations. In this manner that entity comes to be (i) the sole constituent of the so-called "ultimate constituents of matter" and (ii) the formless possessor of all existing physical properties. Observe that, on this view, matter and space are co-implicates; neither exists without the other.

The word "form" has two senses that are relevant here. Forms in the wider sense are sets of relative positions in more or less stable configurations. They come into existence through aggregation—the formation of more or less stable composite objects. Because these forms exist in multidimensional configuration spaces, and because they are probability distributions, they cannot be visualized (or cannot be visualized *except as* multidimensional probability distributions).

Forms in the narrower sense are those that can be visualized *as they are*. The smallest visualizable structure consists of the mean relative positions of a molecule's constituent nuclei—the sticks of the chemist's balls-and-sticks models of molecules. What makes this structure visualizable is the fact that the fuzziness of the relative positions of the nuclei (as measured, for example, by the standard deviations of the corresponding probability distributions) is generally small compared to the mean values of these relative positions.

This, in brief, is how the world of material forms—forms that can be visualized as they are (rather than as probability distributions only)—emerges.

1.16 Assigning Reality

So far we have not made any reality assignment. We only refused, guided by our interpretational strategy, to assign physical reality to a mathematical formalism, to an intrinsically and completely differentiated spacetime manifold, or to a multitude of intrinsically distinct constituents. But we also learned that physical properties need to be realized (made real) before they are available for attribution, and that this explains the special status of measurements: the function of a measurement apparatus—in the broadest sense—is *primarily* to make properties available for attribution and only secondarily to warrant their attribution by indicating their possession (Sec. 1.9). Since a measurement apparatus has itself physical properties, this seems to entail a vicious regress. If the buck is to stop, some properties must be different.

There appears to be only one choice left. It is to assign physical reality to the *macroworld*, defined as the totality of macroscopic positions (short for "positions of macroscopic objects"). Macroscopic objects, so their definition (Sec. 1.14) implies, follow trajectories that are only *counterfactually* fuzzy: their fuzziness does not evince itself in the actual world. Macroscopic positions are fuzzy only in relation to an imaginary background that is more differentiated spacewise than is the actual world. So there is a sense in which *all* positions are fuzzy, and there is a sense in which macroscopic position—and only these—are *not* fuzzy. This goes beyond being not fuzzy "for all practical purposes." A rigorous characterization of macroscopic positions, it permits us to assign *independent* reality exclusively to the macroworld, and to assign dependent reality to whatever can be inferred from property-indicating events ("measurements"), which essentially are unpredictable changes in the values of macroscopic positions.

It deserves to be noted that property-indicating events are unpredictable not only with regard to what they indicate but also with regard to their actual occurrence. Quantum-mechanical probabilities are assigned on the *assumption* that there is an outcome (so that the probabilities of the possible outcomes add up to 1). It is therefore beyond the scope of the quantum theory to provide sufficient conditions for value-indicating events (Ulfbeck and Bohr, 2001; Mohrhoff, 2002a). Quantum mechanics concerns the correlations between value-indicating events that *happen* to occur. All that *we*, as experimenters, can do is maximize the likelihood of occurrence of such an event. This likelihood is not of the kind that quantum mechanics allows us to calculate (i.e., it is not the probability with which this or that outcomes is obtained in a *successful* measurement), but the probability *that* an outcomes is obtained.

1.17 Closing Words

We may turn around and assign independent reality instead to the single persistent entity whose existence we have inferred in Sec. 1.7. As said (Sec. 1.15), by entering into spatial relations with itself, this creates (i) the multitude of spatial relations that constitute space and (ii) the apparent multitude of relata that constitute matter. In the language of the (non-materialist) monistic philosophies of the East as well as the West, it *manifests* the world, or manifests itself as the world.[19] On this view, the quantum theory affords us a glimpse "behind" the manifested world at formless particles and non-visualizable objects, which are *instrumental* in the world's manifestation rather than its constituent parts or structures.

Either way we are dealing with a transition from the indeterminate to the determinate. The so-called microworld sits, as it were, halfway between the indeterminate ground and the determinations that have emerged from it. But what is halfway between no position and a definite position is an indefinite position, and what is halfway between no value and a sharp value is a fuzzy value. But evidence of fuzziness, in the form of a probability distribution over (virtually) definite positions or (virtually) sharp values, cannot be direct. If it is a distribution over *actual* measurement outcomes, then it is evidence of a fuzzy value that *would* have obtained if no measurement had been performed. (As you will remember from Sec. 1.11, fuzzy values are described by distributions over the possible outcomes of *unperformed* measurements.) So it really should not surprise us if we cannot describe what is instrumental in the manifestation of the world except in terms of the finished product—the manifested world.

What is more surprising is the particular form of the quantum-mechanical correlation laws, not least because of their peculiar implications.

[19]It is beyond the scope of this essay to speculate how, and why, this enters into spatial relations with itself, but see Mohrhoff (2008).

But even here one can go a long way toward understanding why they have the particular form that they do. Their validity is guaranteed, for example, by the existence of objects that (i) have spatial extent (they "occupy space"), (ii) are composed of a (large but) finite number of objects without spatial extent ("particles" that do not "occupy space"), and (iii) are stable (they neither explode nor collapse as soon as they are created) (Mohrhoff, 2006b). The self-consistency of these laws, moreover, requires that measurements be possible, and it is eminently plausible that this guarantees the validity of all empirically tested physical theories—the Standard Model and General Relativity—at least as effective theories (Mohrhoff, 2002b). But why are spatially extended objects composed of finite numbers of objects that lack spatial extent? This is perhaps the most intriguing feature of our world.[20]

[20]For a possible answer see Mohrhoff (2008).

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